

Verify the identity.

1. $\csc x - \cot x = \frac{\sin x}{1 + \cos x}$

2. $1 - \frac{\cos^2 x}{1 + \sin x} = \sin x$

3. $\frac{\cos^2 x - \sin^2 x}{\sin x \cos x + \sin^2 x} = \cot x - 1$

4. $\frac{1 + \sin x}{\sin x} + \frac{\cot x - \cos x}{\cos x} = 2 \csc x$

5. $\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$

6. $\sin\left(x + \frac{3\pi}{2}\right) + \sin\left(x - \frac{\pi}{2}\right) = -2 \cos x$

7. $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$

8. $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$

$$9. \sin^2 x + \cos 2x = \cos^2 x$$

$$10. \frac{1+\tan x}{1+\cot x} = \tan x$$

$$11. \cos x - 1 = \frac{\cos 2x - 1}{2 \cos x + 2}$$

$$12. \frac{1+\cos 2x}{\sin^2 x} = 2 \cot^2 x$$

Use an addition or subtraction formula to find the exact value of the expression.

$$13. \sin 255^\circ$$

$$14. \tan \frac{11\pi}{12}$$

Use a half-angle identity to find the exact value of the expression.

$$15. \sin 112.5^\circ$$

$$16. \cos \frac{11\pi}{12}$$

$$17. \tan 75^\circ$$

Find $\sin 2x$, $\cos 2x$, $\tan 2x$ using the given information.

18. $\cot x = \frac{4}{3}$; $\sin x > 0$

19. $\csc x = \frac{25}{7}$; $\tan x < 0$

Find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$, and $\tan \frac{x}{2}$ using the given information.

20. $\cot x = \frac{12}{5}$; $180^\circ < x < 270^\circ$

21. $\cos x = -\frac{8}{17}$; $90^\circ < x < 180^\circ$

Find the primary solution(s) of each equation for $0^\circ \leq \theta < 360^\circ$ or $0 \leq x < 2\pi$. Round your answers to the nearest thousandth when necessary.

22. $4 \cos \theta = -1$

23. $100 \sin \theta = 9 \csc \theta$

24. $\tan^2 x - \tan x - 2 = 0$

25. $\cos x \cot x + 0.1 \cot x = 0$

Find the formulas giving the general solution for each given $0 \leq x < 2\pi$. Your answers must be exact.

26. $\csc^2 x - 4 = 0$

27. $\tan x \sin x + \sin x = 0$

28. $2 \sin x + \sqrt{3} = 0$

29. $\tan^2 x + 2 \tan x + 1 = 0$

30. $\sin x = 1 + \cos^2 x$

31. $\sin 2x + \cos x = 0$

Find all solutions of the equation in the interval $[0, 2\pi)$.

32. $\sec x (2 \cos x - \sqrt{2}) = 0$

33. $2 \sin^2 x - 5 \sin x = -2$

34. $2 \cos^2 x \tan x = \tan x$

35. $2 \sin^2 x - \cos x = 1$

$$36. \tan^2 x + \sec x - 1 = 0$$

$$37. \sin 2x = -\sin x$$

$$38. 3 \tan^2 x - 1 = 0$$

$$39. \cos 2x = \sin^2 x - 2$$

40. In at least one sentence each, explain the difference between each of the following expressions.

$$2 \sin x$$

$$\sin 2x$$

$$\sin^2 x$$

$$\sin \frac{x}{2}$$

Answers:

$$1. -12. \text{ See Online Key}$$

$$13. \frac{-\sqrt{2}-\sqrt{6}}{4}$$

$$14. -2 + \sqrt{3}$$

$$15. \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$16. -\frac{\sqrt{2+\sqrt{3}}}{2}$$

$$17. 2 + \sqrt{3}$$

$$18. \frac{24}{25}, \frac{7}{25}, \frac{24}{7}$$

$$19. -\frac{336}{625}, \frac{527}{625}, -\frac{336}{527}$$

$$20. \frac{5\sqrt{26}}{26}, -\frac{\sqrt{26}}{26}, -5$$

$$21. \frac{5\sqrt{34}}{34}, \frac{3\sqrt{34}}{34}, \frac{5}{3}$$

$$22. \theta = 104.478^\circ; 255.522^\circ$$

$$23. \theta = 17.458^\circ; 162.542^\circ; 197.458^\circ; 342.542^\circ$$

$$24. x = 1.107; x = 4.249; \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$25. x = \frac{\pi}{2}; \frac{3\pi}{2}; 1.671; 4.613$$

$$26. x = \frac{\pi}{6} + \pi k; \frac{5\pi}{6} + \pi k$$

$$27. x = \pi k; \frac{3\pi}{4} + \pi k$$

$$28. x = \frac{4\pi}{3} + 2\pi k; \frac{5\pi}{3} + 2\pi k$$

$$29. x = \frac{3\pi}{4} + \pi k$$

$$30. \frac{\pi}{2} + 2\pi k$$

$$31. x = \frac{\pi}{2} + \pi k; \frac{7\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k$$

$$32. x = \frac{\pi}{4}, \frac{7\pi}{4}$$

$$33. x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$34. 0\pi, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$35. x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$$

$$36. x = 0\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$37. x = 0\pi, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$38. x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$39. x = \frac{\pi}{2}, \frac{3\pi}{2}$$